ESRL Module 8.

Heat Transfer - Heat Recovery Steam Generator Numerical Analysis

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You should complete, or least understand the concepts, in Modules 1 and 4 before trying Module 8.

In this module we examine heat transfer in the heat recovery steam generator (HRSG) in greater detail. For the cogeneration system, changes in process steam demand must be met by the HRSG. The HRSG is typically constructed to meet so called design conditions, but in reality the HRSG must often operate at off-design conditions in order to meet changing steam demand. We examine methods to predict heat recovery equipment performance at off-design conditions and compare these results to process data.

We use numerical methods to determine expected temperature profiles in the HRSG tubes and at the HRSG walls. We will examine heat transfer in the HRSG tubes at steady state using Euler’s method to solve for the temperature profile (an initial value ordinary differential equation). It can also be important to understand the transient temperature response expected from the HRSG. We will introduce this complex topic by using finite differences to solve the unsteady state heat transfer problem in the HRSG wall (an initial value partial differential equation).

**Steady-State Heat Conduction in a One-Dimensional Wall**

We have discussed that energy from the hot exhaust gas exiting the turbine will be recovered in the HRSG – here steam will be generated and cooled combustion products will be vented to the atmosphere from the stack. Energy loss from the walls of the HRSG should be minimal and maximum steam generation within the HRSG should occur. We know that the exhaust gas leaving the gas turbine is about 1390 F (1850 R). Both heat recovery and employee protection dictate that the walls of the transition region between the turbine to the HRSG should be a reasonable temperature. Figure 1 shows the steady-state heat transfer problem. There would be convective heat transfer from the exhaust gas to the wall, conduction through the metal wall, conduction through the insulation and convection from the insulated wall to the room; here we are ignoring any radiation heat transfer considerations. We are considering this problem to be heat transfer in one direction – the temperature is changing in the x direction and uniform in the y- and z-directions.
Figure 1 One-dimension heat transfer with convection and conduction

The convection process can be represented by Newton’s law of heating,

\[ \dot{Q}_{a \rightarrow 1} = h_i A (T_a - T_1) \quad (1) \]

Here the heat transfer rate, \( \dot{Q} \), is given by the temperature difference between the exhaust gas and the inside wall, and the surface area, \( A \), which is taken perpendicular to the direction of heat flow. Similarly from the insulated wall to the room Newton’s law of cooling provides,

\[ \dot{Q}_{3 \rightarrow b} = h_o A (T_3 - T_b) \quad (2) \]

Here \( h_i \) and \( h_o \) are the convection heat transfer coefficients (inside and outside) which are generally determined from available correlations.

The heat transfer rate by conduction can be represented by Fourier’s Law of Heat Conduction, which for the one dimensional heat flow in Figure 1 is,

\[ \dot{Q}_x = -k A \frac{\partial T}{\partial x} \quad (3) \]
where \( k \) is the thermal conductivity of the material. Using equation (3), and applying the forward difference numerical definition for \( \frac{\partial T}{\partial x} \), we can write for the metal wall and the insulation,

\[
\dot{Q}_{1 \rightarrow 2} = -\frac{k_{\text{metal}} A}{\Delta x_{\text{metal}}} (T_2 - T_1) = \frac{k_{\text{metal}} A}{\Delta x_{\text{metal}}} (T_1 - T_2) \tag{4}
\]

\[
\dot{Q}_{2 \rightarrow 3} = \frac{k_{\text{insulation}} A}{\Delta x_{\text{insulation}}} (T_2 - T_3) \tag{5}
\]

At steady state the heat transfer rate through each section must be equal, \( \dot{Q}_{a \rightarrow 1} = \dot{Q}_{1 \rightarrow 2} = \dot{Q}_{2 \rightarrow 3} = \dot{Q}_{3 \rightarrow b} \). Simultaneous solution of these four equations gives,

\[
\dot{Q}_{a \rightarrow b} = \frac{(T_a - T_b)}{\frac{1}{h_i A} + \frac{\Delta x_{\text{metal}}}{k_{\text{metal}} A} + \frac{\Delta x_{\text{insulation}}}{k_{\text{insulation}} A} + \frac{1}{h_o A}} \tag{6}
\]

Or in general for one-dimensional heat transfer,

\[
\dot{Q}_{\text{overall}} = \frac{\Delta T_{\text{overall}}}{\sum R_{\text{thermal}}} \tag{7}
\]

where \( \sum R_{\text{thermal}} \) is the sum of the thermal resistances.

**Example 1 Determining Insulation Thickness** The HRSG (or HRSG transition) in Figure 1 has a \( \frac{1}{4} \) inch carbon steel metal wall with a thermal conductivity, \( k_{\text{metal}} \), of 25 Btu/hr-ft\(^{-1}\)-°F. A glass fiber insulation with a thermal conductivity, \( k_{\text{insulation}} \), of 0.025 Btu/hr-ft\(^{-1}\)-°F is available. If the exhaust gas is 1390 °F, determine the insulation thickness needed to maintain a room-side wall temperature of 125 °F. The inside (exhaust gas to steel wall) convection heat-transfer coefficient, \( h_i \), is 100 Btu/hr-ft\(^2\)-°F and the outside (insulation to room) convection heat-transfer coefficient, \( h_o \), is 1 Btu/hr-ft\(^2\)-°F.

Solution:

The solution to Example 1 is provided in Example 1.xls. We use equation (2) to determine the heat flux at steady state. This heat flux is then used in equation (6) to determine the needed insulation thickness; \( \Delta x_{\text{insulation}} = 6.9 \) in.

**Unsteady-State Heat Conduction in a One-Dimensional Wall**

Unsteady-state heat transfer is needed to predict system response to temperature transients. Unsteady-state heat transfer can be especially important when system physical properties (fluid side or metallurgical) depend on temperature. To account for transients we can write the energy balance,
\[ \text{Rate of heat input} - \text{Rate of heat output} = \text{Rate of heat accumulation} \quad (8) \]

**Unsteady-State Conduction in a Homogeneous Medium**

Consider heat conduction in one-direction, for example though the metal wall of the HRSG, a small section of which is depicted in Figure 2.

\[ \begin{align*}
\Delta y & \quad \Delta z \\
\Delta x & \\
T_x & \quad T_{x+\Delta x}
\end{align*} \]

**Figure 2** Elemental volume for unsteady-state heat conduction in the x-direction

The rate of heat input into and output from the block shown in Figure 2 is given by equation (3),

\[ \text{Rate of heat input} = \dot{Q}_x = -k (\Delta y \Delta z) \left. \frac{\partial T}{\partial x} \right|_x \quad (9) \]

\[ \text{Rate of heat output} = \dot{Q}_{x+\Delta x} = -k (\Delta y \Delta z) \left. \frac{\partial T}{\partial x} \right|_{x+\Delta x} \quad (10) \]

The rate of heat accumulation in the block \((\Delta x \Delta y \Delta z)\) is,

\[ \text{Rate of heat accumulation} = \rho \hat{C}_p (\Delta x \Delta y \Delta z) \frac{\partial T}{\partial t} \quad (11) \]

where \(\rho\) is the density and \(\hat{C}_p\) the heat capacity. Substituting these three equations (9 – 11) into equation (8) and dividing by \((\Delta x \Delta y \Delta z)\) gives,

\[ \left( - \frac{k}{\rho \hat{C}_p} \right) \left( \left. \frac{\partial T}{\partial x} \right|_x - \left. \frac{\partial T}{\partial x} \right|_{x+\Delta x} \right) = \frac{\partial T}{\partial t} \quad (12) \]

Taking the limit as \(\Delta x \to 0\),

\[ \left( \frac{k}{\rho \hat{C}_p} \right) \frac{\partial^2 T}{\partial x^2} = \frac{\partial T}{\partial t} \quad (13) \]

or
where $\alpha = \frac{k}{\rho c_p}$ and $\alpha$ is the thermal diffusivity. Equation (13) (or (14)) relates temperature with time and position, $T(x,t)$, during heat conduction. The analytic solution of equation (13), an initial value PDE, is known for many fixed geometries provided physical properties and boundary conditions are constant with time. The initial temperature profile at time $= 0$ must be known. For example, Figure 1 boundary conditions would include the convection equations with fixed $T_a$ and $T_b$ values for $t > 0$, and the initial temperature profile could be $T(x, 0) = T_b$.

**Unsteady-State Heat Conduction in a Homogeneous Medium – Finite Difference Formulation**

In many situations it is convenient to use numerical methods to solve equation (13). To develop the needed equations we can use temperature nodes as indicated in Figure 3.

![Figure 3 Unsteady state heat conduction at time t.](image)

The conducting body can be divided into blocks of size ($\Delta x \Delta y \Delta z$) as shown in Figure 4, with nodes $1, \ldots, n - 1, n, n + 1$ located at the center of each block.
Using equations (8) – (11), and the forward difference numerical definition for $\frac{\partial \tau}{\partial x}$ and $\frac{\partial \tau}{\partial t}$, we can write the energy balance for block $n$ as,

$$- k (\Delta y \Delta z) \frac{(t \ t_{n} - t \ t_{n-1})}{\Delta x} + k (\Delta y \Delta z) \frac{\Delta x}{\Delta t} \frac{(t \ t_{n+1} - t \ t_{n})}{\Delta x} = \rho \ C_{p} (\Delta x \Delta y \Delta z) \frac{(t + \Delta t \ t_{n} - t \ t_{n})}{\Delta t} \tag{15}$$

Combining terms and assuming constant $k$, $\rho$, $\hat{C}_{p}$ (or thermal diffusivity) gives,

$$\left( \frac{k}{\rho \ C_{p}} \right) \frac{(t \ t_{n+1} + t \ t_{n-1} - 2 t \ t_{n})}{(\Delta x)^{2}} = \left\{ \frac{(t + \Delta t \ t_{n} - t \ t_{n})}{\Delta t} \right\} \tag{16}$$

solving for $t + \Delta t \ t_{n}$,

$$t + \Delta t \ t_{n} = \left( \frac{k \Delta t}{\rho \ C_{p} (\Delta x)^{2}} \right) (t \ t_{n+1} + t \ t_{n-1}) + \left(1 - \frac{2 k \Delta t}{\rho \ C_{p} (\Delta x)^{2}} \right) (t \ t_{n}) \tag{17}$$

Defining $M = \frac{\rho \ C_{p} (\Delta x)^{2}}{k \Delta t}$ equation (17) can be written,

$$t + \Delta t \ t_{n} = \left( \frac{1}{M} \right) (t \ t_{n+1} + t \ t_{n-1}) + \left( 1 - \frac{2}{M} \right) (t \ t_{n}) \tag{18}$$

$M$ must be $\geq 2$. Do note that if $M < 2$, the coefficient of $t \ t_{n}$ in equation (14) becomes negative and values for $t + \Delta t \ t_{n}$ are obtained which violate the second law of thermodynamics. You can choose any combination of $\Delta x$ and $\Delta t$ so $M \geq 2$.

Special Cases:
$M = 2$, the Schmidt Method. When the time and distance increments can be chosen such that,

$$M = \frac{\rho \hat{C}_p (\Delta x)^2}{k \Delta t} = \frac{(\Delta x)^2}{\alpha \Delta t} = 2$$ (19)

the temperature at $t + \Delta t$ $T_n$ becomes,

$$t + \Delta t T_n = \left( \frac{1}{2} \right) \left( t \ T_{n+1} + t \ T_{n-1} \right)$$ (20)

which is simply the arithmetic average of the two adjacent nodes.

$M = 3$ gives,

$$t + \Delta t T_n = \left( \frac{1}{3} \right) \left( t \ T_{n+1} + t \ T_{n} + t \ T_{n-1} \right)$$ (21)

### Unsteady-State Heat Conduction at a Nonhomogenous Boundary – Finite Difference Formulation

Figure 3 shows the plane wall is a composite wall consisting of metal and glass fiber insulation. The thermal diffusivity may be taken as constant in both sections, but we need to account for the interface between the two layers. Using Figure 3 we can imagine that block $n$ represents the perfect contact between the two materials; $A = \text{metal wall}$, $B = \text{glass fiber insulation}$. Equation (15) can be written,

$$- k_A (\Delta y \Delta z) \frac{(t \ T_n - t \ T_{n-1})}{\Delta x} + k_B (\Delta y \Delta z) \frac{(t \ T_{n+1} - t \ T_n)}{\Delta x} =$$

$$\left( \rho_A \hat{C}_{p,A} \frac{\Delta x}{2} + \rho_B \hat{C}_{p,B} \frac{\Delta x}{2} \right) (\Delta y \Delta z) \frac{(t + \Delta t \ T_n - t \ T_n)}{\Delta t}$$ (22)

Notice that the accumulation term consists of the average density and heat capacity of the two materials. In the accumulation term we are approximating that $t \ T_{n-0.5} \approx t \ T_n \approx t \ T_{n+0.5}$. Solving for $t + \Delta t \ T_n$ gives,

$$t + \Delta t \ T_n = \left( \frac{2 \ k_A \Delta t}{(\rho_A \hat{C}_{p,A} + \rho_B \hat{C}_{p,B}) (\Delta x)^2} \right) (t \ T_{n-1})$$

$$+ \left( \frac{2 \ k_B \Delta t}{(\rho_A \hat{C}_{p,A} + \rho_B \hat{C}_{p,B}) (\Delta x)^2} \right) (t \ T_{n+1})$$

$$+ \left( 1 - \left[ \frac{2 \ (k_A + k_B) \Delta t}{(\rho_A \hat{C}_{p,A} + \rho_B \hat{C}_{p,B}) (\Delta x)^2} \right] \right) (t \ T_n)$$ (23)
Unsteady-State Heat Convection Boundary – Finite Difference Formulation

For the one-dimensional system in Figure 1 we still need to account for the convection boundaries at the exhaust gas metal wall interface and the insulation room interface. For the convection boundary shown in Figure 5 we can write the energy balance on the ½ block as,

$$Q_{\text{conv}} = -h(\Delta y \Delta z) (t_{T_a} - t_{T_1}) + k(\Delta y \Delta z) \frac{\Delta T_1}{\Delta t}$$

$$= \rho \hat{C}_p \left(\frac{\Delta x \Delta y \Delta z}{2}\right) \left(\frac{t_{T_2} - t_{T_1}}{\Delta t} - \frac{\Delta x}{2}\right)$$

$$= \rho \hat{C}_p \left(\frac{\Delta x \Delta y \Delta z}{2}\right) \left(\frac{\Delta x}{2}\right)$$

$$= \rho \hat{C}_p \left(\frac{\Delta x \Delta y \Delta z}{2}\right) \left(\frac{\Delta x}{2}\right)$$

Figure 5 Heat conduction at convection boundary

$$h(\Delta y \Delta z) (t_{T_a} - t_{T_1}) + k(\Delta y \Delta z) \frac{\Delta T_1}{\Delta t} = \rho \hat{C}_p \left(\frac{\Delta x \Delta y \Delta z}{2}\right) \left(\frac{\Delta x}{2}\right)$$

$$= \rho \hat{C}_p \left(\frac{\Delta x \Delta y \Delta z}{2}\right) \left(\frac{\Delta x}{2}\right)$$

Solving for $t + \Delta t T_1$ gives,

$$t + \Delta t T_1 = \left(\frac{2 h \Delta t}{\rho \hat{C}_p (\Delta x)}\right) t_{T_a} + \left(1 - \left[\frac{2 h \Delta t}{\rho \hat{C}_p (\Delta x)} + \frac{2 k \Delta t}{\rho \hat{C}_p (\Delta x)^2}\right]\right) (t_{T_1})$$

$$+ \frac{2 k \Delta t}{\rho \hat{C}_p (\Delta x)^2} (t_{T_2})$$

Here the environment is at a temperature $T_a$ and we are neglecting heat accumulation in the exhaust gas (a front ½ block - which is not shown). If $T_a$ varies with time a new value for $t_{T_a}$ can be used at each time step.

With $M = \frac{\rho \hat{C}_p (\Delta x)^2}{k \Delta t}$ and $N = \frac{h(\Delta x)}{k}$

$$t + \Delta t T_1 = \left(\frac{2 N}{M}\right) t_{T_a} + \left(1 - \left[\frac{2 N}{M} + \frac{2}{M}\right]\right) (t_{T_1}) + \frac{2}{M} (t_{T_2})$$

Here we require that $M \geq 2N + 2$, where $N$ is the Nusselt number for this problem and $M$ is the inverse Fourier number.
In a series of example problems we want to first examine heating a metal wall with convection on both surfaces. We then examine the case for the metal wall with a convective surface and a perfectly insulated surface. Then we examine the HRSG composite wall problem of Figure 3.

**Example 2 Heating a Metal Wall with Two Convective Surfaces** Consider the heat conduction problem shown in Figure 6. The ¼ inch carbon steel metal wall has a thermal conductivity, \( k_{metal} = 25 \text{ Btu/hr-ft-}^\circ\text{F} \), \( \rho_{metal} = 500 \text{ lb/ft}^3 \) and \( C_{p,metal} = 0.12 \text{ Btu/lb-}^\circ\text{F} \). Solve for the time dependent metal wall temperature if the wall is initially at 70 F and then exposed to exhaust gas at 1390 F at both exterior surfaces. The convective heat-transfer coefficient on both exterior surfaces, \( h \), is 100 Btu/hr-ft\(^2\)-F.

![Figure 6](image)

**Solution:**
The solution to Example 2 is provided in [Example 2.xls](Example 2.xls). We use equation (26) for the convective surfaces and equation (21) for the conduction in the metal. As \( M \geq 2N + 2 \) it is convenient to set \( M = 3 \) and use \( \Delta x = 0.05 \) inches to create six temperature nodes. Here then \( \Delta t = \left( \frac{500 \text{ lb/ft}^3}{25 \text{ Btu/hr-ft-}^\circ\text{F}} \right) \left( 0.12 \text{ Btu/lb-}^\circ\text{F} \right) \left( \frac{0.05 \text{ F}}{12} \right)^2 = 1.3889 \text{ E-05 hrs (0.05 sec)} \). At 20 seconds the temperature 0.05 inches from the surface is 841.5 F.

**Example 3 Heating a Metal Wall with One Convective Surface and a Perfectly Insulated Surface** The ¼ inch carbon steel metal wall of the HRSG has a thermal conductivity, \( k_{metal} = 25 \text{ Btu/hr-ft-}^\circ\text{F} \), \( \rho_{metal} = 500 \text{ lb/ft}^3 \) and \( C_{p,metal} = 0.12 \text{ Btu/lb-}^\circ\text{F} \). Solve for the time dependent metal wall temperature if the wall is initially at 70 F and then exposed to exhaust gas at 1390 F at one surface and the other surface is perfectly insulated. The convection heat-transfer coefficient, \( h \), is 100 Btu/hr-ft\(^2\)-F. The simplifying assumption of perfect insulation or zero heat flux at a metal wall surface is often made. Zero heat flux is equivalent to setting \( h = 0 \).
Solution:

Accounting for the perfectly insulated wall as shown in Figure 7.

\[ \hat{Q}_x \]

\[ \Delta y \]

\[ \Delta z \]

\[ \Delta x \]

\[ \frac{\Delta x}{2} \]

\[ \text{perfectly insulated wall} \]

Figure 7 Heat conduction at perfectly insulated wall.

Here the energy balance on the ½ block would be,

\[ -k (\Delta y \Delta z) \frac{(t \Delta y - t \Delta y)}{\Delta x} = \rho \ \tilde{c}_p \ \frac{(\Delta x \Delta y \Delta z)}{2} \frac{(t \Delta x - t \Delta x)}{\Delta t} \]

(27)

and solving for \( t + \Delta t \ T_n \),

\[ t + \Delta t \ T_n = \frac{2 k \Delta t}{\rho \ \tilde{c}_p (\Delta x)^2} (t \ T_n - t \ T_{n-1}) + \left( 1 - \frac{2 k \Delta t}{\rho \ \tilde{c}_p (\Delta x)^2} \right) (t \ T_n) \]

(28)

or

\[ t + \Delta t \ T_n = \frac{2 M}{M} (t \ T_{n-1}) + \left( 1 - \frac{2 M}{M} \right) (t \ T_n) \]

(29)

The solution to Example 3 is provided in Example 3.xls. We use equation (26) for the convective surface, equation (21) for the conduction in the metal and equation (29) for the insulated surface. In the Excel solution we set \( M = 3 \) and \( \Delta x = 0.05 \) inches giving \( \Delta t = 1.3889 \ E-05 \) hrs (0.05 sec). At 20 seconds the temperature 0.05 inches from the convective surface is 544.3 F and 0.05 inches from the insulated surface the temperature is 523.3 F.

Example 4 Heating a Composite Wall with Two Convective Surfaces  The HRSG wall shown in Figure 1 is a composite wall consisting of carbon steel and a perfectly contacted insulating glass fiber. The ¼ inch carbon steel metal wall of the HRSG has a thermal conductivity, \( k_{metal} = 25 \) Btu/hr-ft-°F, \( \rho_{metal} = 500 \) lb/ft³ and \( \tilde{c}_{p,metal} = 0.12 \ Btu/lb-°F \). The 6.9 inch glass fiber wall (see Example 3 for thickness) has \( k_{insulation} = 0.025 \) Btu/hr-ft-°F, \( \rho_{insulation} = 5 \) lb/ft³ and \( \tilde{c}_{p,insulation} \)
= 0.2 Btu/lb-°F. The inside (exhaust gas to steel wall) convection heat-transfer coefficient, \( h_i \), is 100 Btu/hr-ft\(^2\)-°F and the outside (insulation to room) convection heat-transfer coefficient, \( h_o \), is 1 Btu/hr-ft\(^2\)-°F. Solve for the time dependent wall temperature if the wall is initially at 70 F and then exposed to exhaust gas at 1390 F. At exposure time = 60 seconds, determine the temperature at the metal / insulation interface, and 0.15 inches from the interface in both the metal and insulation.

Solution:
The solution to Example 4 is provided in [Example 4.xls](#). We use equation (26) for the convective surfaces, equation (21) for the conduction in the metal and insulating fiber and equation (23) for the interface between the metal wall and the fiber. We again use a value of \( M = 3 \) and \( \Delta x = 0.05 \) in. and \( \Delta t = 1.3889 \times 10^{-5} \) hrs (0.05 sec). At 60 seconds the temperature at the interface is 1017.2 F and 0.15 inches from the interface the metal-side temperature is 1026.5 F and the insulation-side temperature is 699.5F.

Here we have examined solution of initial value PDEs using the explicit finite difference method. The method is explicit in that nodes at the current time can be determined using information from nodes at the previous time interval. To keep \( M \) at allowed values the explicit method often necessitates use of a small time step. This problem can be overcome using implicit methods with the Crank-Nicholson method being widely used (Chapra and Canale, 2010).

**Steady-State Heat Conduction in the HRSG**

We have discussed heat transfer considerations in walls typical of the transition from the gas turbine to the HRSG. We now want to examine heat transfer in the boiler tubes. In the HRSG, condensate returning from the process enters the HRSG and is heated to steam by exchanging energy with the hot exhaust gas leaving the turbine. As the water to steam phase change occurs, physical properties which control the effectiveness of the heat transfer will also change. Here we want to show how numerical methods can be used to account for changing system properties and the effects on the heat transfer.

The HRSG water / steam tubes may have a horizontal or more typically a vertical tube arrangement. In an idealized case, as shown in Figure 8, we can consider the HRSG as a cross-flow heat exchanger with water / steam on the tube side and exhaust gas flowing outside the tubes.
Figure 8  a.) HRSG as a cross flow heat exchanger  b.) end view of evaporator  c.) HRSG economizer vertical configuration end-view  d.) side-view

Figure 9a shows the representation of the evaporator section for use in log-mean temperature difference calculations. The evaporator (as well as the economizer) is considered a counter-current
heat exchanger. For example, comparing Figure 9a and Figure 9b, $T_{h,2} = T_g; T_{h,1} = T_{7p}; T_{c,2} = T_g;$ and, $T_{c,1} = T_{8p}$. Here we are following the standard convention that the exhaust gas is a hot ($h$) stream and the water/steam stream is a cold ($c$) stream. The use of $h$ and $c$, as opposed to exhaust_gas and water/steam, helps keep the equations less cumbersome.

For model development a single heat exchanger tube can be considered with Figure 9c serving as a basis for our approach to account for changing physical properties. Figure 9c represents the view normal to the flowing exhaust gas. The temperature of the gas around any section of a vertical tube is uniform with $T_{h(r,n-1)} = T_{h(r,n)} = T_{h(r,n+1)} = \ldots$; where $r$ is the row number (or tube pass number) in the tube bank and $n$ is the node indicator in each tube.

![Diagram](image)

**Figure 9a** Temperature profile in the HRSG evaporator section.
**Figure 9b** Heat Recovery Steam Generator with expanded pinch point
The cold-side equations (c = cold = Water / Steam)

At node $n$ and assuming steady state, we can write the incremental energy transfer rate through $dA$ on the cold (water / steam) side as,

$$\dot{Q}_{(r,n)} = F_c \, d\hat{h}_c(r,n) = F_c \left( \hat{h}_c(r,n+1) - \hat{h}_c(r,n) \right)$$

(30)

Here the cold fluid flow direction is taken as positive and a forward difference is used for $d\hat{h}_c(r,n)$.

The hot-side equations (h = hot = Exhaust Gas)
The temperature of the exhaust gas surrounding any tube in a given row is assumed uniform with 
\[ T_h(r,n-1) = T_h(r,n) = T_h(r,n+1) = \ldots \quad (n = 1, 2, \ldots, N) \]

Heat transfer between hot and cold sides

The incremental heat transfer rate can be expressed as,
\[
\dot{Q}_{(r,n)} = U_{(r,n)} (dA) \left( T_h(r,n) - T_c(r,n) \right) \quad (31)
\]

In equation (31) \( dA \) must be chosen small enough to allow accurate system and physical property determination in the determination of \( U_{(r,n)} \).

If the cold-fluid heat capacities are constant through the volume of area \( dA \), we can substitute for equation (30),

\[
\dot{Q}_{(r,n)} = F_c \left[ \dot{h}_c_{(r,n)} \right] = F_c \left[ \dot{h}_c_{(r,n)} \right] \left( dT_c(r,n) \right)
\]

\[
= F_c \left( \dot{c}_p c_{(r,n)} \right) \left( T_c(r,n+1) - T_c(r,n) \right) \quad (32)
\]

Numerical Solution Strategy – Euler’s Method (for initial value ODE)

Assume all physical and system properties at \( r = 1 \) and \( n = 1 \) are known (initial values). The numerical solution of Figure 9 can be constructed as follows:

1.) Fix an appropriate length \( L \) in the heat exchanger which in turn fixes \( dA \) as \( dA = A_o dL \); \( A_o \) is the external surface area per foot length of tube.

2.) Calculate \( U_{(r,n)} \) which based on the outside surface area of the water/steam tube is,

\[
U_{(r,n)} = \frac{1}{A_i h_{i(r,n)}} + \frac{A_o \ln \left( \frac{d_o}{d_i} \right)}{2 \pi k_{wall} (dL)} + \frac{1}{h_o(r,n)} \quad (33)
\]

In equation (33) \( i \) = inside and \( o \) = outside.

3.) Calculate \( \dot{Q}_{(r,n)} \) using equation (31).

4.) Calculate \( T_c(r,n+1) \) using equation (30) or equation (32). Equation (30) should be used when a phase changes is occurring.

5.) After all cold-side tube calculations are completed \( (n = N) \) calculate \( \dot{Q}_r = \sum_{n=1}^{N} \dot{Q}_{(r,n)} \).

6.) Using \( \dot{Q}_r \) calculate the hot stream temperature for the next row of tubes \( T_h(r+1,n=1..N) \).

This Numerical Solution Strategy will be shown in Example 7.
The numerical solution requires accurate calculation of the inside and outside heat transfer coefficients, \(h_{i,(r,n)}\) and \(h_{o,(r,n)}\) for use in equation (33). Here we must first have knowledge of the HRSG actual physical design in order to determine fluid velocities in the HRSG. We show how the physical design calculations can be performed in the next example.

Before we leave this Numerical Solution Strategy do show yourself that equation (32) and (31) can be combined to give the explicit equation,

\[
T_{c,(r,n+1)} = T_{c,(r,n)} + \frac{U_{(r,n)}}{F_{c}} \left( \bar{c}_{p,c,(r,n)} \right) \left( T_{h,(r,n)} - T_{c,(r,n)} \right) (A_{o} \, dL)
\]

and

\[
T_{h,(r,n+1)} = T_{h,(r,n)}
\]

This is Euler’s Method for \(\frac{dT_{c,(r,n)}}{dL} \neq 0\) and \(\frac{dT_{h,(r,n)}}{dL} = 0\). In the Numerical Solution Strategy we have broken Euler’s Method for \(T_{c,(r,n+1)}\) into two steps which is easier to code (Example 7); see also Module Problem P1.

**Example 5 Determination of the HRSG Physical Design**  For the HRSG of Figure 10 (below) assume the evaporator is 30 tubes wide and the economizer is 20 tubes wide. The tubes are: O.D. = 2 in.; wall thickness = 0.095 in.; length = 15 ft.; tube spacing 4 in. square. Tube data are also provided in Table 1 Using a typical value for overall heat transfer coefficient based on the tube outside area of 12.3 Btu/hr-ft²-°F determine the equivalent number of rows needed in the evaporator and economizer sections.

**Table 1 Evaporator and Economizer Tube Data**

<table>
<thead>
<tr>
<th>Outside Diameter, (d_{o}) (in.)</th>
<th>Inside Diameter, (d_{i}) (in.)</th>
<th>Wall Thickness (in.)</th>
<th>Internal Cross Sectional Area for Flow (in²)</th>
<th>External Surface Area per foot length, (A_{o}) (ft²)</th>
<th>Internal Surface Area per foot length, (A_{i}) (ft²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1.81</td>
<td>0.095</td>
<td>2.573</td>
<td>0.5236</td>
<td>0.4739</td>
</tr>
</tbody>
</table>

Solution:

**Evaporator Section**

Using data from Figure 10,
Stack

Exhaust Gas

Economizer

Evaporator

\[ F_{\text{steam}} = 30.86 \, \text{lb/s} \]

\[ T_o = 874 \, R \]

\[ P_o = 20 \, \text{bar} \]

\[ 233.424 \, \text{lb/s} \]

\[ T_o = 1304.806 \, R \]

\[ P_o = 1.066 \, \text{bar} \]

\[ T_s = 536.67 \, R \]

\[ P_s = 20 \, \text{bar} \]

\[ 30.86 \, \text{lb/s} \]

\[ 9742.04 \, \text{Btu/s} \]

\[ 25981.98 \, \text{Btu/s} \]

\[ \Delta T_{\text{Finch}} = 3R \]

\[ \Delta T_{\text{Approach}} = 27 \, R \]

**Figure 10** HRSG design with \( \Delta T_{\text{Finch}} = 3R \) and \( \Delta T_{\text{Approach}} = 27 \, R \) (real fluid properties)

\[
\frac{dQ_{\text{Evaporator,steam}}}{dt} = \dot{Q}_{\text{Evaporator,steam}} = F_{\text{steam}} \left( \hat{h}_o - \hat{h}_{8P} \right) =
\]

\[
\dot{Q}_{\text{Evaporator,steam}} = 30.86 \, \frac{\text{lb}}{s} \left( 1203.41 - 361.48 \, \frac{\text{Btu}}{\text{lb}} \right) \left( \frac{3600 \, \text{s}}{\text{hr}} \right)
\]

\[
= 93,535,055 \, \frac{\text{Btu}}{\text{hr}}
\]

\[
\Delta T_{\text{LMTD}} = \frac{(T_6 - T_9) - (T_{7P} - T_{8P})}{\ln \left( \frac{(T_6 - T_9)}{(T_{7P} - T_{8P})} \right)} = \frac{430.8 - 30}{\ln \left( \frac{430.8}{30} \right)} = 150.425 \, F
\]

Next calculate the total area needed for the evaporator section using \( \dot{Q} = UA \Delta T_{\text{LMTD}} \),

\[
A = \frac{\dot{Q}}{U \Delta T_{\text{LMTD}}} = \frac{93,535,055 \, \frac{\text{Btu}}{\text{hr}}}{12.3 \, \frac{\text{Btu}}{\text{hr-ft}^2-F}} \left( 150.425 \, F \right) = 50,553 \, \text{ft}^2
\]

Determine the total number of tubes needed for the evaporator section,
\[ N_{\text{tubes, total}} = \frac{A}{(\text{external surface area})} (\text{tube length}) = \frac{A}{(A_o)(L_{\text{tube}})} \]

\[ = \frac{50,553 \text{ ft}^2}{0.5236 \frac{\text{ft}^2}{\text{ft}}} (15 \text{ ft}) = 6435 \text{ tubes} \]

Determine the number of rows needed,

\[ N_{\text{rows, evaporator}} = \frac{\text{number tubes}}{\text{number of tubes wide}} = \frac{6435 \text{ tubes}}{30 \text{ tubes wide}} = 214.46 \text{ rows} \]

The equivalent of about 215 rows is needed in the economizer. The number of rows in the evaporator would be substantially reduced (generally by a factor of 5 or more) if finned tubes as opposed to bare tubes are used.

**Economizer Section**

Again using data from Figure 10,

\[ \dot{Q}_{\text{Economizer, water}} = F_{\text{water}} \left( \frac{\dot{h}_{8P} - \dot{h}_B}{361.48 - 45.79} \right) \frac{Btu}{\text{lb}} \left( \frac{3600 \text{ s}}{\text{hr}} \right) = 35,071,896 \frac{Btu}{\text{hr}} \]

\[ \Delta T_{\text{LMTD}} = \frac{(T_8 - T_7) - (T_{7P} - T_{8P})}{\ln \left( \frac{T_8 - T_7}{T_{7P} - T_{8P}} \right)} = \frac{174.524 - 30}{\ln \left( \frac{174.524}{30} \right)} = 82.075 \text{ F} \]

Next calculate the total area needed for the economizer section using \( \dot{Q} = UA \Delta T_{\text{LMTD}}, \)

\[ A = \frac{\dot{Q}}{U \Delta T_{\text{LMTD}}} = \frac{35,071,896}{12.3} \frac{Btu}{\text{hr} - \text{ft}^2 - \text{F}} (82.075 \text{ F}) = 34,741 \text{ ft}^2 \]

Determine the total number of tubes needed for the economizer section,

\[ N_{\text{tubes, total}} = \frac{A}{(\text{external surface area})} (\text{tube length}) = \frac{A}{(A_o)(L_{\text{tube}})} \]

\[ = \frac{34,741 \text{ ft}^2}{0.5236 \frac{\text{ft}^2}{\text{ft}}} (15 \text{ ft}) = 4423.4 \text{ tubes} \]
Determine the number of rows needed,

\[
N_{\text{Rows,Economizer}} = \frac{\text{number tubes}}{\text{number of tubes wide}} = \frac{4423.4 \text{ tubes}}{20 \text{ tubes wide}} = 221.2 \text{ rows}
\]

The equivalent of about 222 rows is needed in the economizer. This economizer is large (large number of rows) as the condensate return temperature of 77 °F indicated in Figure 10 is low. Generally condensate is returned to the HRSG at temperatures > ~ 200 °F to help prevent acid condensation.

Using the economizer configuration shown in Figure 8, the water to the economizer will be fed to 20 tubes. Each of the 20 tubes will then have ~ 222 passes through the exhaust gas. The nominal length of each tube (222 x 15ft) = 3330 ft. However, accounting for all pipe bends the equivalent length of each tube is about 7400 ft. and the pressure drop would be about 12 psi (Geankoplis, 2003).

**Determination of the inside and outside heat transfer coefficients**

We can now determine the inside and outside heat transfer coefficients for the cross-flow HRSG.

**Outside Heat Transfer Coefficient, \( h_o \) (Btu/hr-ft²-°F)**

The outside heat transfer coefficient is a function of the exhaust gas properties, outside tube diameter and velocity often expressed in terms of the Nusselt (\( Nu \)), Prandtl (\( Pr \)) and Reynolds (\( Re \)) numbers. Ganapathy (1991, 2003) provides a conservative estimate for the outside heat transfer coefficient, \( h_o \), in HRSG applications (exhaust gas external to tubes) as,

\[
h_o = 0.9 G_h^{0.6} d_o^{0.4}
\]

where \( P_h \) = a function of exhaust gas physical properties; \( P_h = \frac{(k_h^{0.6}) (\bar{c}_{p,h}^{0.33})}{\mu_h^{0.27}} \)

\( G_h \) = exhaust gas mass velocity, lb/hr-ft²

\( d_o \) = outside tube diameter, in.

\( k_h \) = exhaust gas thermal conductivity, Btu/hr-ft-°F

\( \bar{c}_{p,h} \) = exhaust gas specific heat, Btu/lb-°F

\( \mu_h \) = exhaust gas viscosity, lb/hr-ft

We can fit \( P_h \) to typical exhaust gas data, over the temperature range typically found in HRSG (200 to 1200 °F), giving

\[
P_h = 3.512 \times 10^{-5} T_{film} + 0.088
\]

where \( T_{film} \) is the film temperature, °F and \( T_{film} \) is the average temperature of the exhaust gas and the tube wall. For example, from Figure 9c, \( T_{film} (r,n) = \frac{T_h (r,n) + T_{wall} (r,n)}{2} \). \( T_{wall} (r,n) \) can be estimated as,
Generally it is sufficient to estimate \( T_{film} (r,n) \) as,

\[
T_{film} (r,n) = \frac{T_h (r,n) + T_c (r,n)}{2}
\] (37)

We can determine \( h_o \) from equation (34) using equation (35) and (37) once a value for the exhaust gas mass flow velocity, \( G_h \) (lb/hr-ft\(^2\)) has been determined. Here

\[
G_h = \frac{F_h}{(N_{tubes,wide}) (L_{tube}) (S_{tube} - d_o) \left( \frac{1}{12 \text{ in.}} \right)}
\] (38)

Where \( F_h \) = the exhaust gas flow rate, lb/hr
\( N_{tubes,wide} = \) the number of tubes wide
\( L_{tube} = \) tube length, ft.
\( S_{tube} = \) tube spacing, in.
\( d_o = \) outside tube diameter, in.

For \( h_o \) we are only considering convective heat transfer. If thermal radiation is significant, the total outside heat transfer coefficient would be the sum of \( h_{o,convection} + h_{o,radiation} \). Generally, \( h_{o,radiation} \) can be neglected for gas temperatures below 800-900 F.

**Inside Heat Transfer Coefficient, \( h_i \) (Btu/hr-ft\(^2\)-°F)**

The inside heat transfer coefficient is a function of the water / steam properties, inside tube diameter and velocity often expressed in terms of the Nusselt (\( Nu \)), Prandtl (\( Pr \)) and Reynolds (\( Re \)) numbers. Generally for turbulent flow in smooth pipes the Dittus Boelter equation (Holman, 1972) is used,

\[
Nu = 0.23 \, Re^{0.8} \, Pr^{0.4} \Rightarrow \quad h_i = 2.44 \, \frac{w_c^{0.8} k_c^{0.6} \hat{C}_{p,c}^{0.4}}{d_i^{1.8} \mu_c^{0.4}}
\] (39)

where \( w_c = \) water /steam flow rate in the single tube, lb/hr
\( d_i = \) inside tube diameter, in.
\( k_c = \) water /steam thermal conductivity, Btu/hr-ft-°F
\( \hat{C}_{p,c} = \) water / steam specific heat, Btu/lb-°F
\( \mu_c = \) water / steam viscosity, lb/hr-ft

The inside heat transfer coefficient is determined using water or steam or two phase properties at the bulk fluid temperature (\( T_c (r,n) \)).
For hot water flowing inside tubes \((T_c(r,n) < 300^\circ F)\), Ganapthy (1991, 2003) provides,

\[
h_l = (150 + 1.55 T_c(r,n)) \frac{v_c^{0.8}}{d_i^{0.2}}
\]

(40)

where \(T_c(r,n)\) = the water temperature, °F
\(v_c\) = the water velocity in the tube, ft/sec
\(d_i\) = the inside tube diameter, in.

**Example 6 Determination of \(h_o, h_l\) and \(U\) for \((r = 1, n = 1)\) in the Economizer**  
Using data from the HRSG economizer of Figure 10 determine \(h_o\) \((r = 1, n = 1)\) and \(h_l\) \((r = 1, n = 1)\).  
Also determine \(U\) \((r = 1, n = 1)\), based on the tube outside surface area, if \(k_{wall} = 25\) Btu/hr-ft-°F.  
Here \(r\) and \(n\) follow the nomenclature of Figure 9c and \(r = 1, n = 1\) would be the first node in the economizer row before the exhaust gas exits the stack.  
Do note that any consistent numbering of the rows (here tube passes) can be used and here \(r = 1, \ldots, N_{rows\_economizer}\).  
Allow that \(T_h\) \((r = 1, n = 1) = T_h\) \((r = 1, n = 2) = T_h\) \((r = 1, n = 3) \ldots = 711.194^\circ R\ (251.524^\circ F)\); this would also be true for any of the first 20 tubes \((N_{tubes\_wide} = 20)\) in the economizer.  
The cold stream enters at \(T_c\) \((r = 1, n = 1) = 536.67^\circ R\ (77^\circ F)\).

**Solution:**

Outside Heat Transfer Coefficient, \(h_o\) (Btu/hr-ft²-°F)

From equation (37)

\[
T_{film(r=1,n=1)} = \frac{T_h(r,n) + T_c(r,n)}{2} = \frac{251.524 + 77}{2} = 164.262 \, ^\circ F
\]

From equation (38)

\[
G_r = \frac{F_h}{(N_{tubes\_wide}) (L_{tube}) (S_{tube} - d_o) \left(\frac{1\, ft}{12\, in.}\right)} = \frac{840.328\, \frac{lb}{hr}}{20\, \text{in.}} = \frac{16806.56\, \frac{lb}{hr}}{f t^2}
\]

From equation (35)

\[
P_h = 3.512 \times 10^{-5} T_{film} + 0.088 = 3.512 \times 10^{-5} (164.262) + 0.088 = 0.0938
\]

From equation (34)

\[
h_o\ (r = 1, n = 1) = 0.9 \frac{G_r^{0.6}}{d_o^{0.4}} = 0.9 \frac{16806.56^{0.6}}{2^{0.4}} = 21.944 \frac{Btu}{hr - ft^2 - ^\circ F}
\]
Inside Heat Transfer Coefficient, \( h_i \) (Btu/hr-ft\(^2\)-°F)

Here we need the velocity of water in the tube at conditions of 536.67°R (77°F) and 20 bar (290.08 psia).

\[
F_{c/tube} = \frac{F_c}{N_{tubes\,wide}} = \frac{111,096 \text{ lb/hr}}{20 \text{ tubes wide}} = 5554.8 \text{ lb/hr - tube}
\]

\[
v_c = \frac{F_{c/tube}}{\pi \left( \frac{d_i}{2} \frac{1\text{ ft.}}{12\text{ in.}} \right)^2 \rho_c} \left( \frac{1\text{ hr}}{3600\text{ sec}} \right) = 0.05093 \frac{F_{c/tube}}{(d_i)^2 \rho_c} = 1.386 \text{ ft/sec}
\]

where \( v_c \) = the water velocity in the tube, ft/sec
\( d_i \) = the inside tube diameter, in.
\( \rho_c \) = water density 77°F, 20 bar (for \( r = 1, n = 1 \)) = 62.3 lb/ft\(^3\)

Using equation (40)

\[
h_i (r = 1, n= 1) = (150 + 1.55 T_c (r,n)) \frac{v_c^{0.8}}{d_i^{0.2}} = \frac{310.214}{hr - ft^2 - ^\circ\, F}
\]

From equation (33)

\[
U_{(r=1,n=1)} = \frac{1}{A_0} \frac{A_0 \ln \left( \frac{d_o}{d_i} \right)}{2 \pi k_{wall} (dL)} + \frac{1}{h_{o (r,n)}}
\]

\[
= \frac{1}{0.5236 \left( \frac{0.4739}{(310.214)} \right) + \frac{0.5236 \ln \left( \frac{2}{1.81} \right)}{2 \pi (25) (1 \text{ ft.})} + \frac{1}{21.944}} = 20.216 \frac{Btu}{hr - ft^2 - ^\circ\, F}
\]

Do note that \( U_{(r=1,n=1)} \sim (0.95) (h_{o (r = 1,n=1)}) \).

**Example 7 Steady-State Temperature Profile for the Economizer Tube Passes**

Using data from the HRSG economizer of Figure 10 develop the numerical solution for the temperature profile in the first two tube passes of the economizer. Use the thermodynamic library for physical properties for the exhaust gas and water. Then solve for the temperature profile in the entire economizer and determine the needed number of tube passes. From Example 6 the number of
passes for each tube was estimated at 222 but here the overall $U$ was assumed to be 12.3 Btu/hr-ft$^2$-°F.

Solution:

The solution to Example 7 is provided in Example 7.xls and the results are shown in Figure 11. The solution in the Excel file can be outlined as follows,

1.) Pick an appropriate $dL$, in the provided Excel code $dL = 1$ ft. but this value can be changed.

2.) Calculate $h_o$, $h_i$, and $U$ as in Example 6 and for the water density use $\rho_c = (1 / (V_{Water}(Pressure_c, T_c(r, n) + 459.67))) \times \text{MW}_{water}$. 

3.) Use equation (31), $\dot{Q}_{(r,n)} = U_{(r,n)} (A_o \times dL) \left( T_h(r,n) - T_c(r,n) \right)$ to calculate the heat transfer rate into the node.

4.) Using equation (30), calculate the total enthalpy into the node as $H_{c\text{ in}} = (H_{Water}(Pressure_c, T_c(r, n) + 459.67)) / \text{MW}_{water} \times F_{c/tube}$. Calculate the total enthalpy out of the node as $H_{c\text{ out}} = H_{c\text{ in}} + \dot{Q}_{(r,n)}$. And determine the temperature of the next node as $T_c(r, n + 1) = T_{HP Water}(H_{c\text{ out}} / F_{c/tube} \times \text{MW}_{water}, \text{Pressure}_c) - 459.67$.

5.) Continue until all the nodes in the current tube pass are solved. Determine the total $\dot{Q}_r$ for the row as $\dot{Q}_r = \sum_{n=1}^{N} \dot{Q}_{(r,n)}$.

6.) Solve for the temperature of the exhaust gas surrounding the $r + 1$ tube pass. First determine the enthalpy of the current exhaust gas in row $r$ using, $H_{Products} \times F_h/N_{tubewide}$. Add $\dot{Q}_r$ to this value. The temperature of the exhaust in row $r + 1$ can then be found using $T_{from\times Products}$. 


Figure 11 Temperature profile in the first two tube passes of the economizer

A simple modification of the Excel file indicates that ~132 tube passes would be necessary to bring $T_c$ to ~387.33°F and here $T_h$ ~ 417.33°F; this results is provided in the Excel file Example 7b.xls.

Acknowledgements

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References


**Student Assignments**

**You are responsible for Problem 1.**

**Problem 1. Steady-State Temperature Profile Single-Pass Countercurrent Double Pipe Heat Exchanger**

Develop a numerical strategy, for the hot and cold temperature profiles in a countercurrent double pipe heat exchanger, which accounts for changing physical properties. A single countercurrent double pipe heat exchanger and temperature profile are indicated in Figures P1a and b. For model development purposes, the single heat exchanger can be considered a series of connected heat exchangers nodes each of area $dA$ as shown in Figure P1c. Figure P1c serves as a basis for our numerical approach to account for changing physical properties in each of the connected heat exchangers. Here Fluid 1 is taken as the hot stream or hot side (the stream giving up energy) and Fluid 2 is assumed the cold stream or cold side (the stream accepting energy).
Figure P1a  Single-pass double pipe heat exchanger.

Figure P1b  Single-pass double pipe heat exchanger - temperature profile.
**Solution Strategy:**

The cold-side equations \((c = cold)\)

At node \(n\) and assuming steady state, we can write the incremental energy transfer rate through \(dA\) on the cold side as,

\[
\dot{Q}_n = F_c \, d\hat{h}_{c,n} = F_c \, (\hat{h}_{c,n+1} - \hat{h}_{c,n}) \quad (P1a)
\]

Here the cold fluid flow direction is taken as positive and a forward difference is used for \(d\hat{h}_{c,n}\).

The hot-side equations \((h = hot)\)

At node \(n\) and assuming steady state, we can write the energy transfer rate through \(dA\) on the hot side as,

\[
\dot{Q}_n = F_h \, d\hat{h}_{h,n} = F_h \, (\hat{h}_{h,n+1} - \hat{h}_{h,n}) \quad (P1b)
\]
Our normal convention follows heat given up by the hot side is a negative quantity as in \( \dot{Q} = F_h \left( \hat{h}_{out} - \hat{h}_{in} \right) \) where out and in are given by the hot fluid flow direction. In equation (P1b), \( \dot{Q}_n \) is a positive quantity because we are indexing by the nodes.

**Heat transfer between hot and cold sides**

The heat transfer rate can also be expressed as,

\[
\dot{Q}_n = U_n (dA) (T_{h,n} - T_{c,n}) \quad (P1c)
\]

In equation (P1c) \( dA \) must be chosen small enough to allow accurate system and physical property determination in the determination of \( U_n \).

If the hot and cold fluid heat capacities are constant through the volume of area \( dA \), we can substitute for equations (P1a) and (P1b),

\[
\dot{Q}_n = F_c d\hat{h}_{c,n} = F_c \left( \hat{C}_{P,c,n} \right) (d T_{c,n}) = F_c \left( \hat{C}_{P,c,n} \right) (T_{c,n+1} - T_{c,n}) \quad (P1d)
\]

and

\[
\dot{Q}_n = F_h d\hat{h}_{h,n} = F_h \left( \hat{C}_{P,h,n} \right) (d T_{h,n}) = F_h \left( \hat{C}_{P,h,n} \right) (T_{h,n+1} - T_{h,n}) \quad (P1e)
\]

**Numerical Solution Strategy**

Assume all physical and system properties at \( n = 1 \) are known, the numerical solution for the temperature profiles (Figure P1) can be constructed as follows:

1.) Fix an appropriate length \( dL \) in the heat exchanger which in turn fixes \( dA \) as \( dA = A_o \, dL; A_o \) is the external surface area per foot length of tube.

2.) Calculate \( U_n \), which for the double pipe heat exchanger of Figure P1 which based on the inside surface area of the fluid 2 tube is,

\[
U_n = \frac{1}{\frac{1}{h_{i,n}} + \frac{A_i \ln \left( \frac{d_o}{d_i} \right)}{2 \pi k_n (dL)} + \frac{A_i}{A_o} \frac{1}{\hat{h}_{o,n}}} \quad (P1f)
\]

The value for \( U_n \) based on the outside area of the fluid 2 tube can be found using equation (33).

3.) Calculate \( \dot{Q}_n \) using equation (P1c).

4.) Calculate \( T_{c,n+1} \) using equation (P1d) and calculate \( T_{h,n+1} \) using equation (P1e).

Here we have a system of two ordinary differential equations which are solved using Euler’s Method.
\[ T_{c,n+1} = T_{c,n} + \frac{U_n}{F_c \left( \hat{c}_p_{c,n} \right)} \left( T_{h,n} - T_{c,n} \right) (A_0 \, dL) \]
\[ T_{h,n+1} = T_{h,n} + \frac{U_n}{F_h \left( \hat{c}_p_{h,n} \right)} \left( T_{h,n} - T_{c,n} \right) (A_0 \, dL) \]

Two initial conditions at the starting \( dL \) value \( (n = 1) \) are required. If, for example, the hot stream properties are only known at the hot inlet, \( n = N \), guess \( T_{h,1} \) and solve until the cold stream outlet temperature is reached. \( T_{h,1} \) can be varied until the correct hot stream inlet temperature is obtained.

**Student Assignments - Laboratory Course**

You must complete the Student Assignment (above) before attempting this laboratory course student assignment.

**Laboratory Course – Student Assignment** Go to the web site [www.esrl.lsu.edu](http://www.esrl.lsu.edu) or [www.cogened.lsu.edu](http://www.cogened.lsu.edu) and access operational data from the LSU cogeneration facility found in the folder *Cogeneration Operational Data*.

LSU is evaluating installation of a heat exchanger to pre-heat the natural gas being fed to the combustion chamber. The current plan calls for part of the steam from the heat recovery steam generator (HRSG) to be used for this purpose. Design this heat exchanger using the numerical model you developed in this module (Problem 1).

A young engineer has also suggested, that instead of using steam, the heat exchanger could be installed in the exhaust stack from the system (after the heat recovery steam generator). Do you think this alternative suggestion should also be evaluated?